PRICE STABILIZATION OF TWO-STAGE STOCHASTIC PROGRAMS WITH APPLICATION TO ENERGY GENERATION PROBLEMS

Clara Macedo Lage - ENGIE - Panthéon Sorbonne University - IMPA - (CIFRE)
Claudia Sagastizábal - Unicamp
Mikhail Solodov - IMPA
Guillaume Erbs - ENGIE

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Modern energy markets involve a large number of units and different technologies to generate electricity.

In Brazil and Northern Europe, hydraulic generation is one of the main sources of energy. This technology has two important characteristics:

- The low cost of hydro-energy generation if compared to others sources.
- The difficulty in predicting the amount of rain or snow at any time scale makes the water inflows uncertain.

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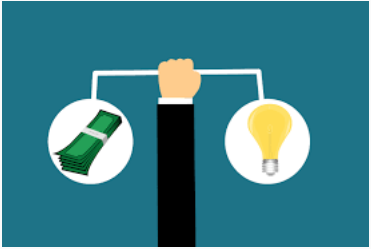
Because of the low cost of hydro-energy generation, its price is defined considering the possibility of shortage of energy and the prices of others sources.



For example, if it rains less than expected, it can be necessary to activate different and more expensive power plants. This extra-cost is an important component in the price of hydro power plants.

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The randomness that comes from the diverse inflow scenarios makes us consider many different possibilities in a future cost function.

We consider scenarios with large and small amount of inflow and its consequences for the system.

Denoting the generation of the i-th unit by x_i, a stylized model for optimizing the overall generation is given by:

$$\begin{aligned} \min \quad & \langle \text{Cost}, x \rangle \\ \text{s.t.} \quad & \textstyle\sum_i x_i = \text{demand} \\ & x_i \leq \text{Capacity}_i \\ & x_i \geq 0 \end{aligned}$$

A signal for the energy prices is obtained from the multiplier of the demand constraint. In the model, demand is uncertain (due to the water inflow).

The components of the Lagrange multiplier vector π^B give the rates of change of the optimal value with respect to perturbations in the demand.

$$\begin{aligned} \min \quad & \langle \mathrm{Cost}, x \rangle \\ \text{s.t.} \quad & \textstyle\sum_i x_i = \mathrm{demand} \;\rightsquigarrow\; \pi^B \text{ (Price)} \\ & x_i \leq \mathrm{Capacity} \\ & x_i \geq 0 \end{aligned}$$

The stylized problem is one block in an equilibrium model when all the agents in the market aim at maximizing their revenue.

In long term planning problems, decisions are coupled in time. An example of the link between t and $t+1$ is the water balance equation.

For one realization ξ of the uncertain Inflow, the 2-stage formulation is:

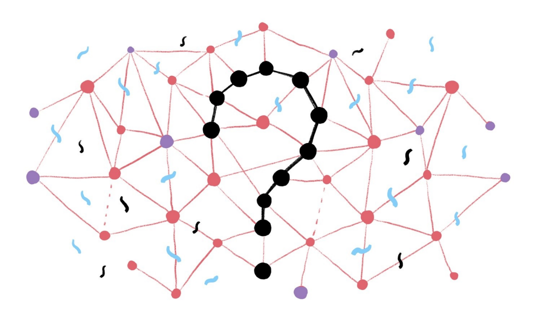
$$\begin{array}{ll}\min & \langle \mathrm{Cost}_1, x_1\rangle + \langle \mathrm{Cost}_2, x_2\rangle \\ \text{s.t.} & x_i \geq 0, i = 1,2 \\ & Bx_i = b_i, i = 1,2 \\ & Tx_1 + Wx_2 = \mathrm{Inflow}_\xi \ \rightsquigarrow \ \pi(\xi)\end{array}$$

The link between stages is represented by the matrices W and T.

The sub-vectors x_1 and x_2 represent, respectively, the parameters in the generation of the set of power plants, at time steps 1 and 2.

Variables x_2 are recourse variables that depend on the realization ξ.

The set of Lagrange Multipliers $\{\pi\}$ is commonly singleton. The price signal is a choice in this set that depends on the way we model and the algorithm used to solve the problem.



Is this price signal the best one to our application? What do we know about the position of this price signal in this set?

Also, taking uncertainty into account means that the price will be a random vector. It means that price distribution changes depending on the demand distribution or rain distribution over a month or a year.

We can observe it in this example that was made with a bunch of problems in stochastic optimization.

Table: Price comparison (sample size = 200).

Sample	$E[\pi_1]$	$E[\pi_2]$
1	0.79	-2.013
2	1.40	-1.568
3	0.87	-1.959

We aim at defining a mechanism to stabilize the price signals w.r.t. sample variations

MATHEMATICAL MODEL

In the two stage model, for each realization ξ of the uncertainty, the price is given by the Lagrange Multiplier of the corresponding second stage problem.

First Stage Problem:

$$\begin{array}{ll} \min & \langle \mathrm{Cost}_1, x_1 \rangle + E[Q(x_1, \xi)] \\ \text{s.t.} & x_1 \geq 0 \\ & B_i x_i \leq b_i \end{array}$$

Second Stage Problem, fixed ξ_i:

$$Q(x_1, \xi_i) := \begin{cases} \min & \langle \mathrm{Cost}_2, x_2 \rangle \\ \text{s.t.} & W x_2 = \mathrm{inflow}_{\xi_i} - T x_1 \\ & B_2 x_2 \leq b_2 \\ & x_2 \geq 0 \end{cases}$$

The second stage, also known as future cost function:

$$Q(x_1, \xi) := \begin{cases} \min & \langle \text{Cost}_2, x_2 \rangle \\ \text{s.t.} & Wx_2 = \text{inflow}_{\xi_i} - Tx_1 \\ & B_2 x_2 \leq b_2 \\ & x_2 \geq 0 \end{cases}$$

Has as dual:

$$Q(x_1, \xi) := \begin{cases} \max & \langle \pi, \text{inflow}_{\xi_i} - Tx_1 \rangle - \left\langle b_2, \pi^B \right\rangle \\ \\ \text{s.t.} & W^\intercal \pi - B_2^\intercal \pi^B \leq \text{Cost}_2 \end{cases}$$

Where π^B is the Lagrange Multiplier of the inequality constrain.

We are specially interested in the Price Signal π, the Lagrange Multiplier of the water balance constraint.

Lagrange Multiplier (Water Balance Constraint):

$$\Pi^i(x) = \{\pi : \exists\, \pi^B \text{ s.t. } (h_{\xi_i} - Tx)^T\pi - \langle b_2, \pi^B \rangle = Q(x, \xi_i)\}$$

The Lagrange multiplier π, can also be viewed as an element of the sub-gradient of Q :

$$\pi(\xi) \in \{g \ : \ Q(z, \xi) \geq Q(x, \xi) + \rangle g, z - x\}$$

Consider the second stage problem.

Given a constant $\beta > 0$, the regularized second stage problem is:

$$Q^\beta(x_1, \xi) := \begin{cases} \max & \langle \pi, \text{inflow}_\xi - Tx_1 \rangle - \left\langle b_2, \pi^B \right\rangle - \frac{\beta}{2}\|\pi\|^2 \\ \text{s.t.} & W^T \pi \leq \text{Cost}_2 \end{cases}$$

or, computing its dual:

$$Q^\beta(x_1, \xi) = \begin{cases} \min & \langle \text{Cost}_2, x_2 \rangle + \frac{1}{2\beta}\|\text{inflow}_\xi - Tx_1 - Wx_2\|^2 \\ \text{s.t.} & x_2 \geq 0 \\ & B_2 x_2 \leq b_2 \end{cases}$$

Our goal is to study the primal-dual dual solution when $\beta \to 0$.

THEORETICAL RESULTS

To simplify notation, we denote Cost_i as F_i and Inflow as h, considering **h** deterministic.

The first stage (QP) problem is:

$$\min_{x_1 \geq 0} \langle F_1, x_1 \rangle + Q^{\beta}(x_1)\,,$$

and has the following one-level formulation:

$$\begin{cases} \min & \langle F_1, x_1 \rangle + \langle F_2, x_2 \rangle + \frac{1}{2\beta}\|h - Tx_1 - Wx_2\|^2 \\ \text{s.t.} & x_1 \geq 0\,, x_2 \geq 0\,, B_1 x_1 \leq b_1\,, B_2 x_2 \leq b_2\,. \end{cases}$$

Given a primal solution $\bar{x}_1{}^{\beta}$, $\bar{x}_2{}^{\beta}$ the regularized price signal will be:

$$\bar{\pi}^{\beta} = \frac{1}{\beta}(h - T\bar{x}_1{}^{\beta} - W\bar{x}_2{}^{\beta}).$$

Result 1 If the optimization problem has a unique solution $\bar{x}$, and the sequence $\beta_k \to 0$ is non decreasing, the solution x^k of:

$$\begin{cases} \min & \langle F_1, x_1 \rangle + \langle F_2, x_2 \rangle + \frac{1}{2\beta_k} \|h - Tx_1 - Wx_2\|^2 \\ \text{s.t.} & x_1 \geq 0\,, x_2 \geq 0\,, B_1x_1 \leq b_1\,, B_2x_2 \leq b_2\,. \end{cases}$$

is such that:

$$x^k = (x_1^k, x_2^k) \to (\bar{x}_1, \bar{x}_2) = \bar{x}$$

THEORETICAL RESULTS

Our main interest is in the price π. Keeping h fixed, we know that:

$$\pi^k = \frac{1}{\beta_k}(h - Tx_1^k - Wx_2^k).$$

The questions that arise naturally are:

1. Is π^k bounded?
2. Which assumptions are necessary to make π^k bounded?
3. Does π^k converge and how can we interpret the limit.

Our second result proves that the sequence of regularized price signals π^k is bounded under certain conditions, and that it converges to the minimum price of the original problem.

Result 2 Let $\beta_k > \beta_{k+1} \to 0$. Suppose that the original problem has a unique solution $\bar{x}$, and let $\bar{\mu}$ be an optimal multiplier of the constraint $x \geq 0$. As shown before: $x^k \to \bar{x}$ as $k \to \infty$.

There is a subsequence π^{k_j}, that converges to $\hat{\pi}$, the minimum-norm solution Lagrange multiplier, and solution of:

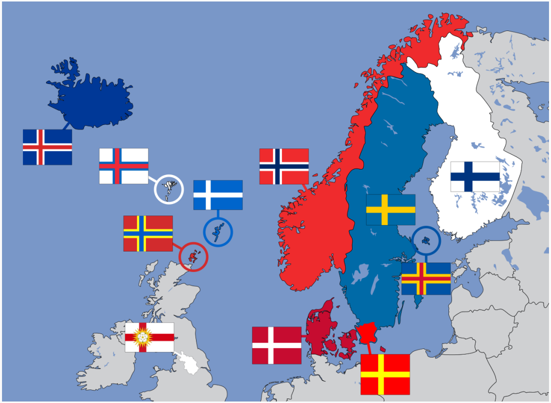
$$\begin{array}{ll} \min\limits_{x} & \|\pi\|^2 \\ \text{s.t} & F + A^T\pi - \bar{\mu} - B^T\bar{\lambda} = 0. \end{array}$$

The point $(\bar{x}, \hat{\pi}, \bar{\mu}, \bar{\lambda})$ is a primal-dual solution of the original problem.

APPLICATION TO ENERGY GENERATION PROBLEM

The Northern Europe generation system is composed by hydro, wind, thermal and solar power plants.

Hydro energy is the most important component in the region.



APPLICATION TO ENERGY GENERATION PROBLEM

Our model of generation has the following characteristics:

- Two Stage Model with 6-month first stage (winter and autumn) and 6-month second stage (spring and summer).
- One water balance (WB) equation per hydro with random inflows.
- Deterministic demand varies with (winter and summer).
- System has 31 geographical zones which may or may not interchange energy (bounded flow).

For the regularized problem it is important to think about the choice of β. Looking at the formula:

$$\bar{\pi}^{\beta} = \frac{1}{\beta}(h - T\bar{x_1}^{\beta} - W\bar{x_2}^{\beta}),$$

and considering that the order of magnitude of the price is known, we conclude that the choice of β determines the tolerance for the balance water equation.

Therefore it is reasonable that: $\bar{\pi}^{\beta} \times \beta \leq \mathrm{Var}(h(\xi))$, since we do not want to allow greater variations in the water balance equation then variations in the scenarios inflows.

In our model, this line of reasoning and some empirical tests, lead us to the choice of $\beta = 1.2$.

Typically, the energy price is given by the most expensive power plant that attends the demand. Things change with the zone exchanges.

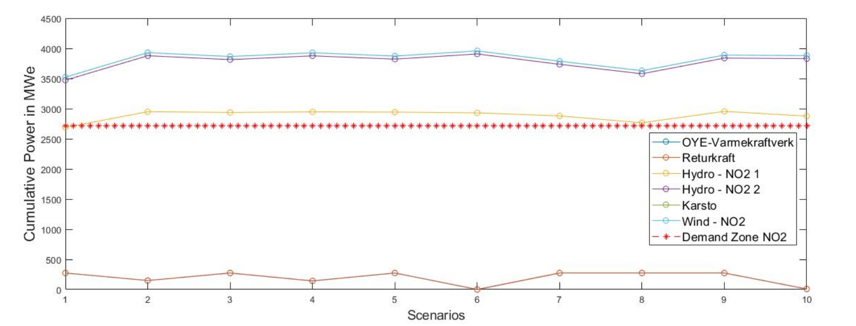
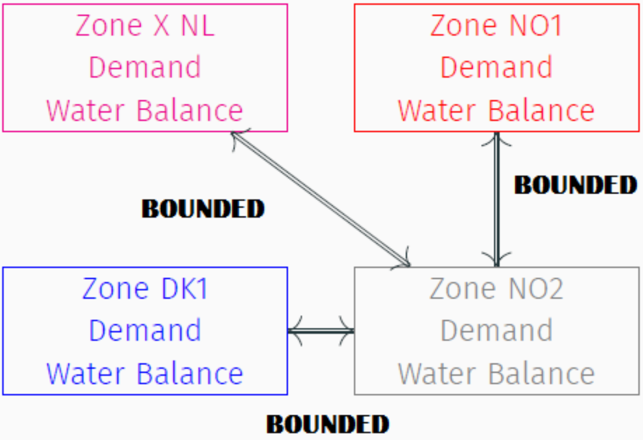


Figure: ZONE NO2 in Norway

Zones are connected but there is a bound "FlowMax(z_1, z_2)" bounding possible exchanges.



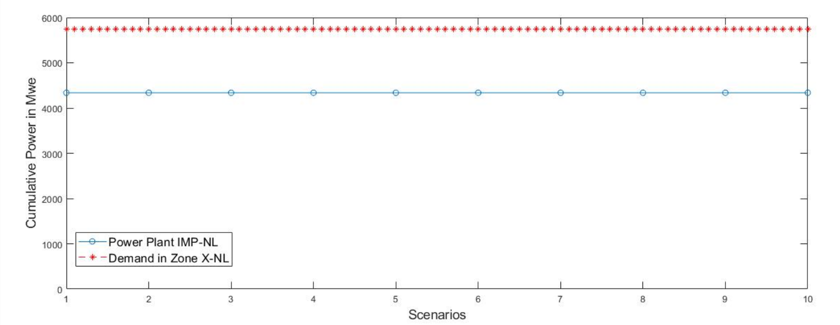


Figure: ZONE X GB

This zone receives the excess of production of zone NO2 to attend its demand. It is a Zone of importation/exportation.

Marginal prices are affected by congested import/export connections”:

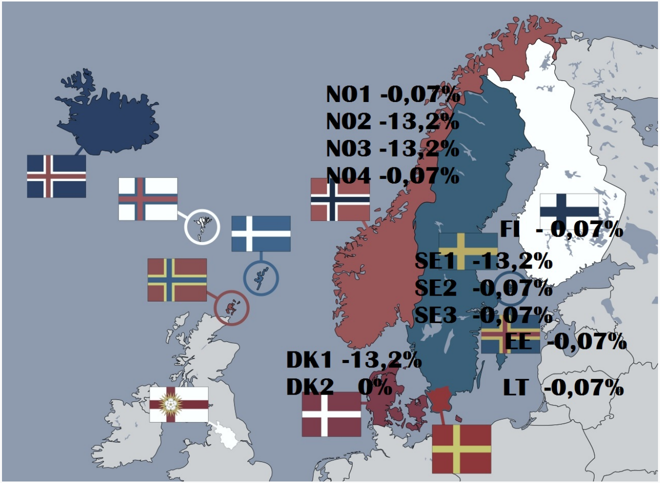
FlowMax and Flow for scenarios 40 and 41

Scenario	Flow between NO2 and X-GB	FlowMax
40	643,95	1400
41	1400	1400
45	1400	1400

Component of Price Signal for scenarios 40 and 41:

Scenario	Price Signal for hydro NO2	Power Plant with this price
40	65,7337	IMP-GB (Zone X-GB)
41	48,93	Kartzo (Zone NO2)
45	34,10	Nordjyllandsvaerket (Zone DK1)

Percentage reduction of price with regularization compared with price without regularization per zone:



Expected Value of Price with and without Regularization:

Zone	Price without Regularization	Price with Regularization
DK1 (Denmark)	249.6659	213.6453
DK2 (Denmark)	156.2775	156.2775
EE (Estonia)	134.6180	133.0853
FI (Finland)	134.6180	133.0862
LT (Lithuania)	134.6180	133.0853
NO1 (Norway)	134.6180	133.0853
NO2 (Norway)	249.6659	213.6453
NO3 (Norway)	249.6659	213.6453
NO4 (Norway)	134.6180	133.0862
NO5 (Norway)	134.6180	133.0862
SE1 (Sweden)	249.6659	213.6453
SE2 (Sweden)	134.6180	133.0862
SE3 (Sweden)	134.6180	133.0862

CONCLUSIONS AND NEXT STEPS

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- The theoretical and numerical results seem to be consistent.
- The results in optimization theory about convergence have made us pay attention to the expectation value for problems in witch more than one solution is possible.
- The next step is to work in the energy generation problem, think about variance and regularization in the application and theory.

Thanks for your attention!