Analysis of EPEC Models for Power Markets

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- At the beginning electricity market were government-owned monopoly, resulting in frequent service interruptions.
- The electricity industry has undergone a restructuring process over the last three decades, which has led to the establishment of wholesale electricity markets.
- The goal is the implementation and development of electricity markets that stimulates competition and market efficiency on behalf of society.
- A short-term electricity market is implemented, where energy transactions are held with the participation of independent power producers.

Currently, Brazil follows the Tight Pool Model.

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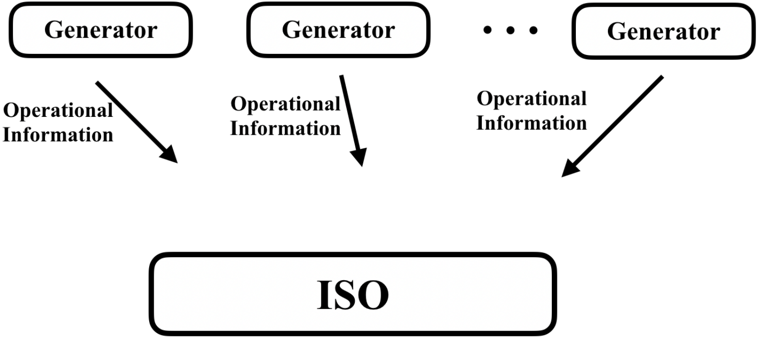

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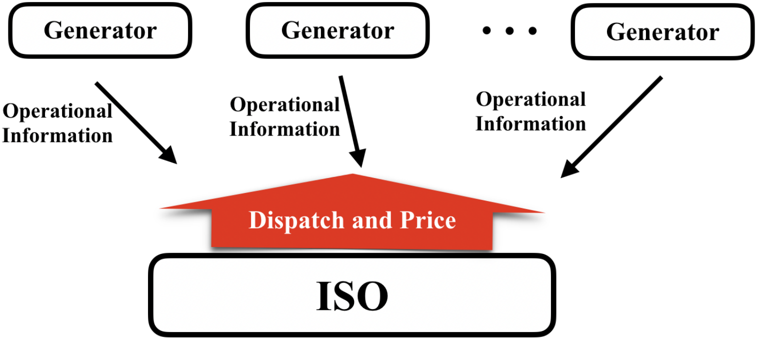

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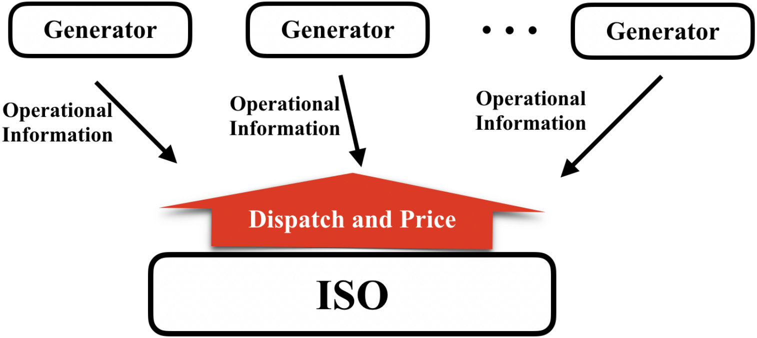

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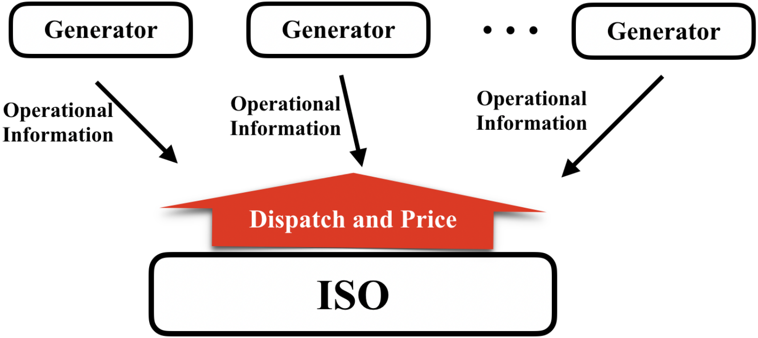

- The policy for price formation is not straightforward.
- Some generators might fail to cover generation costs.

In the future, it may follow a Loose Pool Model.

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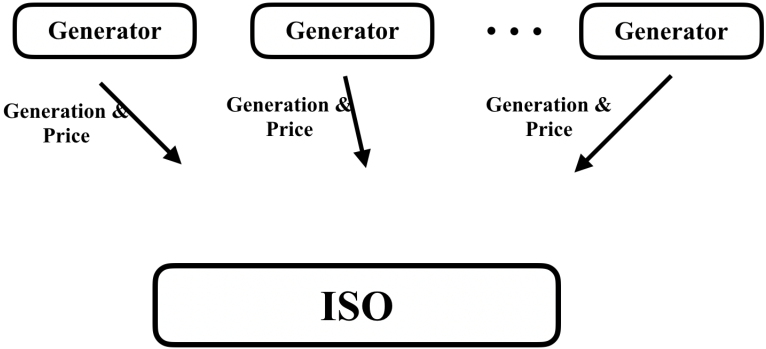

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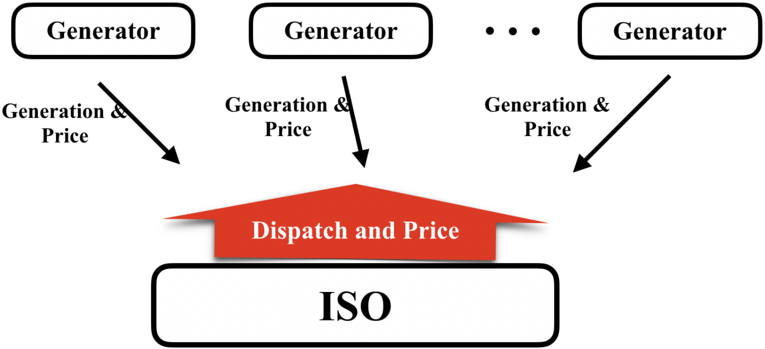

- We aim to determine the producers bid in a short-term electricity market.
- Each generator faces a decision-making problem of estimating the best offer of price and quantity that maximizes their net revenue, taking into account the unknown bid strategy of their opponents.

We consider the case of energy generators participating in a day-ahead electricity market.

- Each generator makes a bid for energy (price and quantity) for each of the 24 hour in the schedule day.
- They maximize revenue considering their opponents decisions and the regulator (ISO) behavior.
- The ISO takes all producer bids and computes the generation dispatch minimizing the system total cost of operation.

- According to the producers nature (e.g. hydroelectric, thermoelectric) their decision may be strongly coupled.
- The resulting model is highly nonlinear and challenging.

Cruz, M. P., Finardi, E. C., de Matos, V. L., & **Luna, J. P.** (2016). **Strategic bidding for price-maker producers in predominantly hydroelectric systems.** Electric Power Systems Research, 140, 435-444.

Generalized Nash Equilibrium Problem (GNEP)

For agent i seeks to solve

$$P_i(x^{-i}) \begin{cases} \min_{x^i} & f^i(x^i, x^{-i}) \\ \text{s.t.} & x^i \in D^i \\ & x^i \in X^i(x^{-i}) \end{cases}$$

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- x^i: agent decision variable.
- x^{-i}: other agents decision variables.

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Nash Equilibrium

A point $\bar{x}$ is a Nash Equilibrium if for each i, $\bar{x}^i$ solves $P_i(x^{-i})$.

The Model

Decision Variable and Objective Function

$x^i = (g^i, p^i, l^i)$, where

- g_i generation bid (MWh)
- p_i price bid (\$/MWh)
- l_i dispatch (MWh)

$$\begin{cases} \min\limits_{p_i, g_i, l_i} & f^i(p_i, g_i, l_i, P(p, g, l)) \\ \text{s.t.} & (p_i, g_i) \in S^i \\ & g \in S^O \\ & (p, g, l) \in S^{ISO} \end{cases}$$

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The Model

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- g_i generation bid (MWh)
- p_i price bid ($/MWh)
- l_i dispatch (MWh)

$$\begin{cases} \min\limits_{p_i, g_i, l_i} & f^i(p_i, g_i, l_i, P(p, g, l)) \\ \text{s.t.} & (p_i, g_i) \in S^i \\ & g \in S^O \\ & (p, g, l) \in S^{ISO} \end{cases}$$

$$f^i(p_i, g_i, l_i, P(p, g, l)) = \varphi^i g^i - P(p, g, l) l^i$$

where φ_i is i-th agent marginal cost.

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The Model

The energy price $P(p, g, l)$ is defined via some policy. An examples is

$$P(p, g, l) := \max \{p_j : l_j > 0\} \ .$$

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$$S^i : \begin{cases} 0 \leq g_i \leq g_i^{\max} \\ \psi_i(g_i) \geq p_i \geq \varphi_i \end{cases}$$

where

- $g_i^{\max}$ maximum generation
- $\psi_i(g_i)$ dynamic (generation-related) bidding rule
  1. ψ_i is a strictly decreasing function of generation
  2. $\psi_i(0) = \kappa_i \varphi_i, \quad \kappa_i \geq 1$
  3. $\psi_i(g_i^{\max}) = \varphi_i$

  $\psi_i(g_i) = \varphi_i \left(1 + (\kappa_i - 1) \frac{g_i^{\max} - g_i}{g_i^{\max}}\right)$

The Model

Coupling Constraints:

Operational Coupling Constraints

Can be expressed as linear equations.

The ISO's Problem

$$S^{ISO} = \left\{ (p, g, l) : l \in \underset{l}{\operatorname{argmin}} \left\{ \begin{array}{ll} p^\top l & \\ 0 \leq l \leq g & \\ \sum_i l_i = d & (\pi) \end{array} \right\} \right\}$$

where d is the demand.

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The Model

$$\begin{aligned}
\min \quad & \varphi_i g_i - P(g,p,l) l_i \\
\text{s.t.} \quad & 0 \le g_i \le g_i^{\max} \\
& \psi_i(g_i) \ge p_i \ge \varphi_i \qquad \text{(1a)} \\
l_i \quad & \in \arg\min \left\{ \begin{array}{c} p_i l_i + p_{-i} l_{-i} : \\ 0 \le l_i \le g_i \\ 0 \le l_{-i} \le g_{-i} \\ l_i + l_{-i} = d \quad (\pi_i) \end{array} \right\} \qquad \text{(1b)}
\end{aligned}$$

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If the bids are such that $0 < p_1 < p_2 < \ldots < p_N$, $g_1\,, g_2\,\ldots g_N > 0$ and $\sum_j g_j \geq d$.

$$\begin{cases} \min & \sum_{j=1}^{N} l_j p_j & \\ \text{s.t.} & 0 \leq l_j \leq g_j & \forall j = 1, 2, \ldots N \\ & \sum_{j=1}^{N} l_j = d & (\leftrightarrow \pi)\,, \end{cases}$$

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$\exists j^*$ unique such that

$$\sum_{j=1}^{j^*-1} g_j < d \leq \sum_{j=1}^{j^*} g_j .$$

The j^*th agent is marginal.

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$$\begin{cases} \min & \sum_{j=1}^{N} l_j p_j \\ \text{s.t.} & 0 \leq l_j \leq g_j \qquad \forall j = 1, 2, \ldots N \\ & \sum_{j=1}^{N} l_j = d \qquad (\leftrightarrow \pi)\,, \end{cases}$$

The solution $\bar{l}$ is given by

$$\bar{l}_j = \begin{cases} g_j, & \text{if } j < j^* \\ d - \sum_{j=1}^{j^*-1} g_j > 0, & \text{if } j = j^* \\ 0 & \text{if } j > j^*\,. \end{cases}$$

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If the bids are such that $0 < p_1 < p_2 < \ldots < p_N$, $g_1\,, g_2\,\ldots g_N > 0$ and $\sum_j g_j \geq d$.

$$\begin{cases} \min & \sum_{j=1}^{N} l_j p_j & \\ \text{s.t.} & 0 \leq l_j \leq g_j & \forall j = 1, 2, \ldots N \\ & \sum_{j=1}^{N} l_j = d & (\leftrightarrow \pi)\,, \end{cases}$$

As for the dual solution, the price $\bar{\pi}$ is given by

$$\begin{cases} \bar{\pi} = p_{j^*} & \text{if} \quad \bar{l}_{j^*} < g_{j^*} & \text{(unique multiplier)} \\ \bar{\pi} \in [p_{j^*}, p_{j^*+1}] & \text{if} \quad \bar{l}_{j^*} = g_{j^*} & \text{(compact set of multipliers)} \\ \bar{\pi} \in [p_{j^*}, +\infty) & \text{if} \quad j^* = N & \text{(unbounded set of multipliers).} \end{cases}$$

Proposition

In the two-agent model, assuming that $0 < \varphi_1 < \varphi_2$, $0 < g_i^{\max} < d$, for $i = 1, 2$, and $g_1^{\max} + g_2^{\max} > d$ we have that

1. The point $\tilde{g}_1 = g_1^{\max}$, $\tilde{p}_1 = \varphi_1$, $\tilde{g}_2 = d - g_1^{\max}$, $\tilde{p}_2 = \psi_2(\tilde{g}_2)$ and $\tilde{\pi} = \tilde{p}_2$ is always an equilibrium for the model.
2. The point $\tilde{g}_1 = d - g_2^{\max}$, $\tilde{p}_1 = \psi_1(\tilde{g}_1)$, $\tilde{g}_2 = g_2^{\max}$, $\tilde{p}_2 = \varphi_2$ and $\tilde{\pi} = \tilde{p}_1$ is an equilibrium for the model, whenever $\psi_1(\tilde{g}_1) > \varphi_2$ and $(\varphi_1 - \psi_1(d - g_2^{\max}))(d - g_2^{\max}) < (\varphi_1 - \varphi_2)g_1^{\max}$.

$$
\begin{aligned}
\min_{p_i, g_i, l_i} \quad & \varphi_i g_i - P(p,g,l) l_i \\
\text{s.t.} \quad & 0 \le g_i \le g_i^{\max} \\
& \psi_i(g_i) \ge p_i \ge \varphi_i \\
l_i \quad & \in \arg\min \left\{ \begin{array}{c} p_i l_i + p_{-i} l_{-i} : \\ 0 \le l_i \le g_i \\ 0 \le l_{-i} \le g_{-i} \\ l_i + l_{-i} = d \quad (\pi_i) \end{array} \right\}
\end{aligned}
$$

$$
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\end{aligned}
$$

$$\pi = P(p, g, l)?$$

$$\begin{array}{rl} \min & \varphi_i g_i - \pi l_i \\ \text{s.t.} & 0 \leq g_i \leq g_i^{\max} \\ & \psi_i(g_i) \geq p_i \geq \varphi_i \\ & KKT(p_i, l, g_i, \pi) \end{array}$$

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Primal

$$(P)\left\{\begin{array}{lll} \min & \sum_{j=1}^{N} l_j p_j & \\ \text{s.t.} & 0 \leq l_j \leq g_j & \forall j = 1, 2, \ldots N \\ & \sum_{j=1}^{N} l_j = d & (\leftrightarrow \pi)\,, \end{array}\right.$$

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Primal

$$(P)\left\{\begin{array}{lll} \min & \sum_{j=1}^{N} l_j p_j & \\ \text{s.t.} & 0 \leq l_j \leq g_j & \forall j = 1, 2, \ldots N \\ & \sum_{j=1}^{N} l_j = d & (\leftrightarrow \pi)\,, \end{array}\right.$$

Dual

$$(D)\left\{\begin{array}{ll} \min & \lambda^T g - \pi d \\ \text{s.t.} & \pi - \lambda_j \leq p_j\,,\ \text{for } j = 1, \ldots, N \\ & \lambda_j \geq 0\,,\ \text{for } j = 1, \ldots, N\,, \end{array}\right.$$

Primal

$$(P)\begin{cases} \min & \sum_{j=1}^{N} l_j p_j \\ \text{s.t.} & 0 \leq l_j \leq g_j \qquad \forall j = 1, 2, \ldots N \\ & \sum_{j=1}^{N} l_j = d \qquad (\leftrightarrow \pi)\,, \end{cases}$$

Penalized Dual

$$(D_\beta)\begin{cases} \min & \lambda^T g - \pi d + \beta \|\lambda\|_\infty \\ \text{s.t.} & \pi - \lambda_j \leq p_j\,, \text{ for } j = 1, \ldots, N \\ & \lambda_j \geq 0\,, \text{ for } j = 1, \ldots, N\,, \end{cases}$$

Primal

$$
(P_\beta)\begin{cases}
\min & \sum_{j=1}^{N} l_j p_j & \\
\text{s.t.} & 0 \le l_j, w_j & \forall j = 1, 2, \dots N \\
 & 0 \le l_j - w_j \le g_j & \forall j = 1, 2, \dots N \\
 & \sum_{j=1}^{N} l_j = d & (\leftrightarrow \pi) \\
 & \sum_{j=1}^{N} w_j = \beta &
\end{cases}
$$

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\text{s.t.} & \pi - \lambda_j \le p_j\,, \text{ for } j = 1, \dots, N \\
 & \lambda_j \ge 0\,, \text{ for } j = 1, \dots, N\,,
\end{cases}
$$

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Proposition

Given a solution $(\pi_\beta, \lambda_\beta)$ we have that $\pi_\beta \geq 0$ and

$$(\lambda_\beta)_j = [\pi_\beta - p_j]^+$$

Also, if $(\pi_\beta^1, \lambda_\beta^1)$ and $(\pi_\beta^2, \lambda_\beta^2)$ are solutions,we have that

- $\pi_\beta^1 \leq \pi_\beta^2$ only when $\lambda_\beta^1 \leq \lambda_\beta^2$.
- $\pi_\beta^1 < \pi_\beta^2$ and $\lambda_\beta^2 \neq 0$, then $\|\lambda_\beta^1\|_\infty < \|\lambda_\beta^2\|_\infty$.

Proposition

Given the family solutions $(\pi_\beta, \lambda_\beta)_{\beta \geq 0}$ we have that

1. The family is bounded.
2. If $(\hat{\pi}, \hat{\lambda})$ is an accumulation point of the family (when $\beta \to 0$), then it is a solution of D_0. Also, if (π_0, λ_0) any other solution of D_0, then $\hat{\pi} \leq \pi_0$ and $\hat{\lambda} \leq \lambda_0$.

$$\boxed{\pi_\beta \approx P(p, g, l)}$$

$$\begin{aligned} \min \quad & \varphi_i g_i - \pi l_i \\ \text{s.t.} \quad & 0 \leq g_i \leq g_i^{\max} \\ & \psi_i(g_i) \geq p_i \geq \varphi_i \\ & KKT(p_i, l, g_i, \pi_\beta) \end{aligned}$$

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Thank You!