DC programming techniques with inexact subproblems' solution for general DC programs resulting from bilevel programs

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PID19, Palaiseau, 16/01/2019

Outline

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Unit-commitment problem

The unit-commitment problem in energy management aims at finding the optimal production schedule of a set of generation units while meeting several system-wide constraints

- Optimal scheduling (next day) of a set of generation units coupled with system-wide constraints
- Declined in many different versions
  - Bilateral or centralized market frameworks
  - System with hydro/thermal/nuclear utilities
  - Intermittent sources (sun and wind)
- Intermittent and run-of-river generation are uncertain
- (Most common) sources of uncertainty
  - Renewable generation (water inflows, wind, sun)
  - Energy demand
  - Unit availability
  - Energy prices

It has always been a large-scale, non-convex difficult problem, especially in view of the fact that operational requirements imply that it has to be solved in an unreasonably small time for its size

Energy management: a new landscape

The energy landscape undergoes significant changes worldwide

- In addition to classical generating companies, large consumers have gained protagonism and can perform actions which may significantly impact the load of the system
- As a consequence of the change in the demand of a large consumer, the load of the network is changed and power gets redistributed in the system (for instance, preventing congestion at peak times)
  - It is possible to shift consumption from one moment of the day to another one deemed more convenient or profitable for the consumer
- Such modifications are globally beneficial for the system because generation costs and constraints are highly nonlinear
- Another important feature of this new setting is the (imminent) use of batteries which partially store intermittent energy and, hence, contribute to mitigating the uncertainty that is inherent to such power systems

Energy management: a new landscape

In order to best represent this new landscape, two new elements must be incorporated in the energy management model of the system

- **Aggregators**: take advantage of regrouping a set of customers without generation assets
- **Microgrids**: deal with nearly isolated systems that handle locally their generation, in the quest for remaining (as much as possible) independent of the global network

In either case, "optimal" interactions between the local actors (aggregators and micro-grids) with classical generators (represented by a global actor) is crucial for the quest of

- higher profitability
- lower CO_2 emissions
- confiability

Energy management: a new landscape

- In this work, we focus on the interaction between a large global actor and a local actor representing a smart grid
- Both actors dispose of several means of power generation, such as solar cells, wind generation, etc...
- The local actor will attempt to meet the local load as well as possible
- The local actor will also interact with the global actor
  - Excess of local power generation can be sold to the global actor
  - The local actor can also buy power from the global actor to meet the local demand
- The global actor is responsible for adjusting the generation level of his assets to meet system-wide load

Energy management: centralized system

Before presenting the bilevel formulation, let's consider a conventional centralized operated system

$$\begin{cases} \min\limits_{z,w} & \sum_{j=1}^{m} f_j(z_j) + h(w) \\ \text{s.t.} & \sum_{j=1}^{m} A^j z_j + Cw = d \\ & w \in \Omega,\ z_j \in Z^j, j = 1, ..., m, \end{cases}$$

- the vector $d \in \mathbb{R}^T$ represents system wide load
- the vectors $z_1, ..., z_m$ are decisions related to generating with assets $i = 1, ..., m$ (nuclear generation, coal, hydro generation)
- w is the demand side management vector
- $Z^1, ..., Z^m$ and Ω the abstract set of constraints associated with these decisions
- $A^i z_i$ (Cw respectively) represent the effect on the amount of generated power of a given decision
- The cost functions $f_1, ..., f_m, h$ are assumed to be convex

In a centralized setting, h and C can be assumed to be zero

Energy management: local actor

Moving away from the centrally operated world, we partition the set $\{1, 2, \ldots, m\}$ of assets (generating units) into two groups

- the assets owned by the global actor: $G = \{1, ..., m_g\}$
- the assets owned by the local actor: $L = \{m_{g+1}, ..., m\}$

Then, decision on generation is represented by $z = (z^G, z^L)$ System wide load is assumed to be $d = d^G + d^L$ When the demand-side-management tools w are at the control of the global actor and interfere with the local offer demand balance, the lower level model is:

$$V(w) := \begin{cases} \min\limits_{z^L, y^e, y^l} & \sum_{i \in L} f_i(z_i) - (c^e)^\top y^e + (c^l)^\top y^l \\ \text{s.t.} & z_j \in Z^i, j \in L \\ & y^l - y^e + \sum_{j \in L} A^i z_i = d^L - Cw \\ & y^e, y^l \in \mathbb{R}^T_+. \end{cases}$$

- Local excess y^e of power generation can be sold to the global actor at price c^e
- Local lack y^l of power generation can be bought from the global actor at price c^l $(> c^e)$

1 Unit-commitment problem
- unit-commitment

2 Mathematical formulation
- Value function approach

3 DC programming
- On optimality
- Algorithms

4 Numerical experiments
- Early experiments - Proof of concept

Energy management: local actor

$$V(w) = \begin{cases} \min_{z^L, y^e, y^l} & \sum_{i \in L} f_i(z_i) - (c^e)^\top y^e + (c^l)^\top y^l \\ \text{s.t.} & z_j \in Z^i, j \in L \\ & y^l - y^e + \sum_{i \in L} A^i z_i = d^L - Cw \\ & y^e, y^l \in \mathbb{R}^T_+. \end{cases}$$

- This problem corresponds to a situation wherein the local actor ignores uncertainty on local load
- However, the global actor is responsible for dealing and accounting for this uncertainty, and has two stages of decision
- In the second stage, the global actor can adjust the generation level of his assets to meet the system-wide load

$$Q^s(y) := \begin{cases} \min_{z^G} & \sum_{j=1}^{m_g} f_j(z_j^s) \\ \text{s.t.} & w \in \Omega,\ z_j^s \in Z^j, j \in G \\ & \sum_{j \in G} A^j z_j + Cw = d_s^G - y^e + y^l \end{cases}$$

Provided $\Omega, Z^j, j = 1, \ldots, m$ are convex sets:

- $V(\cdot)$ is a convex function on variable w
- $Q^s(\cdot)$ is a convex function on variable y

Bilevel formulation

- Second-level subproblem (Local problem)

$$V(w) = \begin{cases} \min\limits_{z^L, y^e, y^l} & \sum_{i \in L} f_i(z_i) - (c^e)^\top y^e + (c^l)^\top y^l \\ \text{s.t.} & z_j \in Z^i, j \in L \\ & y^l - y^e + \sum_{i \in L} A^i z_i = d^L - Cw \\ & y^e, y^l \in \mathbb{R}^T_+. \end{cases}$$

- Second-stage subproblem

$$Q^s(y) = \begin{cases} \min\limits_{z^G} & \sum_{j=1}^{m_g} f_j(z_j^s) \\ \text{s.t.} & w \in \Omega,\ z_j^s \in Z^j, j \in G \\ & \sum_{j \in G} A^j z_j + Cw = d_s^G - y^e + y^l \end{cases}$$

First level problem: global actor

$$\begin{cases} \min\limits_{w,y} & h(w) + (c^e)^\top y^e - (c^l)^\top y^l + \sum_{s \in S} \pi_s Q^s(y) \\ \text{s.t.} & (y^e, y^l) \in \arg\min \text{(Local subproblem)} \end{cases}$$

A stochastic nonsmooth bilevel programming problem

Bilevel problem: a value-function formulation

- Second-level subproblem (Local problem)

$$V(w) = \begin{cases} \min\limits_{z^L, y^e, y^l} & \sum_{i\in L} f_i(z_i) - (c^e)^\top y^e + (c^l)^\top y^l \\ \text{s.t.} & z_j \in Z^i, j \in L \\ & y^l - y^e + \sum_{i\in L} A^i z_i = d^L - Cw \\ & y^e, y^l \in \mathbb{R}^T_+. \end{cases}$$

- Second-stage subproblem

$$Q^s(y) = \begin{cases} \min\limits_{z^G} & \sum_{j=1}^{m_g} f_j(z_j^s) \\ \text{s.t.} & w \in \Omega,\ z_j^s \in Z^j, j \in G \\ & \sum_{j\in G} A^j z_j + Cw = d_s^G - y^e + y^l \end{cases}$$

Global actor

$$\begin{cases} \min\limits_{z,w,y} & h(w) + (c^e)^\top y^e - (c^l)^\top y^l + \sum_{s\in S} \pi_s Q^s(y) \\ \text{s.t.} & \sum_{i\in L} f_i(z_i) - (c^e)^\top y^e + (c^l)^\top y^l - V(w) \leq 0 \\ & z_j \in Z^i, j \in L \\ & y^l - y^e + \sum_{i\in L} A^i z_i + Cw = d^L \\ & y^e, y^l \in \mathbb{R}^T_+. \end{cases}$$

General DC programming problem

It turns out that the considered problem is a *General DC programming problem*[1]

$$\begin{cases} \min\limits_{x} & f_1(x) - f_2(x) \\ \text{s.t.} & c_1(x) - c_2(x) \leq 0 \\ & x \in X, \end{cases}$$

with $f_1, f_2, c_1, c_2 : \mathbb{R}^n \to \mathbb{R} \cup \{\infty\}$ convex functions, and X a convex set

- $x = (z, w, y)$
- $f_1(x) = h(w) + (c^e)^\top y^e - (c^l)^\top y^l + \sum_{s \in S} \pi_s Q^s(y)$ (non-differentiable)
- $f_2(x) = 0$
- $c_1(x) = \sum_{i \in L} f_i(z_i) - (c^e)^\top y^e + (c^l)^\top y^l$
- $c_2(x) = V(w)$ (non-differentiable)

Questions

- What is the definition of a stationary point for a general DC programming problem? (recall that non-differentiable non-convex functions are in general non-regular)
- Can we always compute a stationary point? How?
- Do we need constraint qualification (CQ)?
- What type of CQ?

[1] H.A. Le Thi, H.V. Ngai, and P.D. Tao, DC programming and DCA for general DC programs. Advanced Computational Methods for Knowledge Engineering, 2014, pp. 15-35.

General DC programming: B-stationarity I

$$\left\{\begin{array}{ll} \min_x & f_1(x) - f_2(x) \\ \text{s.t.} & c_1(x) - c_2(x) \leq 0 \\ & x \in X \end{array}\right. \equiv \left\{\begin{array}{ll} \min_x & f_1(x) - f_2(x) \\ \text{s.t.} & x \in X^c \end{array}\right.$$

Let $X^c := \{x \in X : \ c_1(x) - c_2(x) \leq 0\}$ and $\mathcal{T}_{X^c}(\bar{x})$ the Bouligand tangent cone of X^c at point $\bar{x} \in X^c$

In mathematical terms, $d \in \mathcal{T}_{X^c}(\bar{x})$ if there exist a sequence of vectors $\{x^k\} \subset X^c$ converging to $\bar{x}$ and a sequence of positive scalars $\tau_k \to 0$ such that $d = \lim_{k\to\infty}(x^k - \bar{x})/\tau_k$

General DC programming: *B*-stationarity II

Definition: *B*(ouligand)-stationarity [2]

A point $\bar{x} \in X^c$ is called a *B*-stationary point of the DC program above if

$$f_1'(\bar{x}; d) \geq f_2'(\bar{x}; d) \quad \forall\, d \in \mathcal{T}_{X^c}(\bar{x}),$$

where, for $i = 1, 2$, $f_i'(x; d) := \lim_{t \downarrow 0}[f_i(x + td) - f_i(x)]/t$ is the directional derivative of f_i at point x and direction d

How to verify if a given point $\bar{x} \in X^c$ is *B*-stationary?
In other words, how to characterize the Bouligand tangent cone of X^c at point $\bar{x} \in X^c$?

[2] J.S. Pang, M. Razaviyayn, and A. Alvarado, Computing b-stationary points of nonsmooth DC programs, Mathematics of Operations Research 42 (2017), pp. 95-118

General DC programming: B-stationarity III

$$\left\{\begin{array}{ll} \min\limits_x & f_1(x) - f_2(x) \\ \text{s.t.} & c_1(x) - c_2(x) \leq 0 \\ & x \in X \end{array}\right. \equiv \left\{\begin{array}{ll} \min\limits_x & f_1(x) - f_2(x) \\ \text{s.t.} & x \in X^c \end{array}\right.$$

Proposition (W. van Ackooij, W. de Oliveira 2017)

Let $\bar{x} \in X^c$ be such that $c_1(\bar{x}) = c_2(\bar{x})$. Suppose that the following Slater constraint qualification (CQ) holds:

$$\text{there exists } \bar{d} \in \mathcal{T}_X(\bar{x}) \text{ such that } c_1'(\bar{x}; \bar{d}) < c_2'(\bar{x}; \bar{d}).$$

The point $\bar{x}$ is B-stationary point of the above DC program if and only if

$$\bar{x} \in \left\{\begin{array}{ll} \arg\min\limits_x & f_1(x) - [f_2(\bar{x}) + \langle g_2, x - \bar{x}\rangle] \\ s.t. & c_1(x) \leq c_2(\bar{x}) + \langle s_2, x - \bar{x}\rangle \\ & x \in X \end{array}\right. \qquad \begin{array}{l} \forall\, g_2 \in \partial f_2(\bar{x}) \\ \forall\, s_2 \in \partial c_2(\bar{x}) \end{array}$$

This is still not practical unless we assume some structure...

DC programming: particular case

Assume that f_2, $c_2 : \mathbb{R}^n \to \mathbb{R}$ are convex functions defined by the pointwise maximum of finitely many differentiable convex functions, i.e.,

$$f_2(x) := \max_{i=1,\dots,m_f} \psi_i(x), \quad c_2(x) := \max_{i=1,\dots,m_c} \varphi_i(x) .$$

Let $A^f(x) := \{1 \leq i \leq m_f : f_2(x) = \psi_i(x)\}$ and $A^c(x) := \{1 \leq j \leq m_c : c_2(x) = \varphi_j(x)\}$

Corollary

Let $\bar{x} \in X^c$ satisfying $c_1(\bar{x}) = c_2(\bar{x})$ and suppose there exists $\bar{d} \in \mathcal{T}_X(\bar{x})$ such that $c_1'(\bar{x}; \bar{d}) < c_2'(\bar{x}; \bar{d})$. Then $\bar{x}$ is a B-stationary point if and only if

$$\bar{x} \in \left\{ \begin{array}{ll} \arg\min_x & f_1(x) - [f_2(\bar{x}) + \langle \nabla \psi_i(\bar{x}), x - \bar{x} \rangle] \\ s.t. & c_1(x) \leq c_2(\bar{x}) + \langle \nabla \varphi_j(\bar{x}), x - \bar{x} \rangle \\ & x \in X \end{array} \right. \qquad \forall\, i \in A^f(\bar{x}) \text{ and } \forall\, j \in A^c(\bar{x})$$

This result has been proved in [Pang et al.(2017)Pang, Razaviyayn, and Alvarado] under the more restrictive *Pointwise Slater CQ*: there exist $\bar{d}^j \in \mathcal{T}_X(\bar{x})$ satisfying $c_1'(\bar{x}; \bar{d}^j) < \langle \nabla \varphi_j(\bar{x}), \bar{d}^j \rangle$ for all $j \in A^c(\bar{x})$

For more general DC programs, we will need to consider a weaker condition of stationarity, known as criticality

General DC programming: criticality

Let $\bar{x} \in X^c$ satisfying $c_1(\bar{x}) = c_2(\bar{x})$

- Considered Slater CQ:

$$\text{there exists } \bar{d} \in \mathcal{T}_X(\bar{x}) \text{ such that } c_1'(\bar{x};\bar{d}) < c_2'(\bar{x};\bar{d})$$

- *B*-stationarity:

$$\bar{x} \in \begin{cases} \arg\min_x & f_1(x) - [f_2(\bar{x}) + \langle g_2, x - \bar{x}\rangle] \\ \text{s.t.} & c_1(x) \leq c_2(\bar{x}) + \langle s_2, x - \bar{x}\rangle \\ & x \in X \end{cases} \qquad \begin{array}{l} \text{for all } g_2 \in \partial f_2(\bar{x}) \\ \text{for all } s_2 \in \partial c_2(\bar{x}) \end{array}$$

- Criticality:

$$\bar{x} \in \begin{cases} \arg\min_x & f_1(x) - [f_2(\bar{x}) + \langle g_2, x - \bar{x}\rangle] \\ \text{s.t.} & c_1(x) \leq c_2(\bar{x}) + \langle s_2, x - \bar{x}\rangle \\ & x \in X \end{cases} \qquad \begin{array}{l} \text{for an arbitrary } g_2 \in \partial f_2(\bar{x}) \\ \text{for an arbitrary } s_2 \in \partial c_2(\bar{x}) \end{array}$$

If both functions f_2 and c_2 are continuously differentiable, then the concepts of stationarity and criticality coincide

General DC programming: criticality

Let $\bar{x} \in X^c$ satisfying $c_1(\bar{x}) = c_2(\bar{x})$

- Considered Slater CQ:

there exists $\bar{d} \in \mathcal{T}_X(\bar{x})$ such that $c_1'(\bar{x};\bar{d}) < c_2'(\bar{x};\bar{d})$

- *B*-stationarity:

$$\bar{x} \in \begin{cases} \arg\min_x & f_1(x) - [f_2(\bar{x}) + \langle g_2, x - \bar{x}\rangle] \\ \text{s.t.} & c_1(x) \leq c_2(\bar{x}) + \langle s_2, x - \bar{x}\rangle \\ & x \in X \end{cases}$$

for all $g_2 \in \partial f_2(\bar{x})$
for all $s_2 \in \partial c_2(\bar{x})$

- Criticality:

$$\bar{x} \in \begin{cases} \arg\min_x & f_1(x) - [f_2(\bar{x}) + \langle g_2, x - \bar{x}\rangle] \\ \text{s.t.} & c_1(x) \leq c_2(\bar{x}) + \langle s_2, x - \bar{x}\rangle \\ & x \in X \end{cases}$$

for an arbitrary $g_2 \in \partial f_2(\bar{x})$
for an arbitrary $s_2 \in \partial c_2(\bar{x})$

If both functions f_2 and c_2 are continuously differentiable, then the concepts of *B*-stationarity and criticality coincide

Criticality × KKT point

Criticality:

$$\bar{x} \in \begin{cases} \arg\min_x & f_1(x) - [f_2(\bar{x}) + \langle g_2, x - \bar{x} \rangle] \\ \text{s.t.} & c_1(x) \leq c_2(\bar{x}) + \langle s_2, x - \bar{x} \rangle \\ & x \in X \end{cases}$$

for an arbitrary $g_2 \in \partial f_2(\bar{x})$
for an arbitrary $s_2 \in \partial c_2(\bar{x})$

A critical point, when is it a KKT point of the general DC problem?

Lemma

Assume that $\bar{x}$ is a solution of the above subproblem, and that there exists a Slater point $x^\circ \in X$ such that $c_1(x^\circ) < c_2(\bar{x}) + \langle s_2, x^\circ - \bar{x} \rangle$. Moreover, suppose that

- [(a)] either f_1 or f_2 is continuously differentiable, and
- [(b)] either c_1 or c_2 is continuously differentiable

Then, there exists a Lagrange multiplier $\lambda \geq 0$ such that the point $(\bar{x}, \lambda)$ satisfies the KKT system of the general DC programming problem

$$\begin{cases} 0 \in \partial[f_1(\bar{x}) - f_2(\bar{x})] + \lambda(\partial[c_1(\bar{x}) - c_2(\bar{x})]) + N_X(\bar{x}) \\ c_1(\bar{x}) - c_2(\bar{x}) \leq 0 \\ \lambda[c_1(\bar{x}) - c_2(\bar{x})] = 0 \\ \lambda \geq 0, \; \bar{x} \in X \end{cases}$$

DCA - DC Algorithm

Criticality:

$$\bar{x} \in \begin{cases} \arg\min_x & f_1(x) - [f_2(\bar{x}) + \langle g_2, x - \bar{x}\rangle] \\ \text{s.t.} & c_1(x) \le c_2(\bar{x}) + \langle s_2, x - \bar{x}\rangle \\ & x \in X \end{cases}$$

for an arbitrary $g_2 \in \partial f_2(\bar{x})$
for an arbitrary $s_2 \in \partial c_2(\bar{x})$

We may employ a DC algorithm [3], [4] to compute a critical point for the general DC programming problem

DC Algorithm - DCA

- Let $x^0 \in X^c$ be given
- For $k = 0, 1, \ldots$ compute a solution of the following convex program

$$x^{k+1} \in \begin{cases} \arg\min_x & f_1(x) - [f_2(x^k) + \langle g_2^k, x - x^k\rangle] \\ \text{s.t.} & c_1(x) \le c_2(x^k) + \langle s_2^k, x - x^k\rangle \\ & x \in X \end{cases}$$

for an arbitrary $g_2^k \in \partial f_2(x^k)$
for an arbitrary $s_2^k \in \partial c_2(x^k)$

- If $x^{k+1} = x^k$ stop

[3] H.A. Le Thi, H.V. Ngai, and P.D. Tao, DC programming and DCA for general DC programs. Advanced Computational Methods for Knowledge Engineering, 2014, pp. 15-35.

[4] P.D. Tao and H.A. Le Thi, Convex analysis approach to DC programming: theory, algorithms and applications, Acta Mathematica Vietnamica 22 (1997), pp. 289-355.

IDCA: new proposal

DC algorithm with inexact subproblem's solution

- In our application, solving exactly these (DCA) subproblems is too time-consuming
- It amounts to solving a (modified) unit-commitment problem per iteration
- We, therefore, propose to solve such subproblems inexactly

$$Y(x^k) := \{x \in X : \; c_1(x) \le c_2(x^k) + \langle s_2^k, x - x^k \rangle\}$$

DC Algorithm with inexact subproblem's solution

- Let $x^0 \in X^c$ be given
- For $k = 0, 1, \ldots$ find $x^{k+1} \in Y(x^k)$ satisfying either

$$(I) \quad f_1(x^{k+1}) - [f_2(x^k) + \langle g_2^k, x^{k+1} - x^k \rangle] \le f_1(x^k) - f_2(x^k) \quad \text{and} \quad x^{k+1} \ne x^k$$

or

$$(II) \quad g_2^k \in \partial f_1(x^{k+1}) + N_{Y(x^k)}(x^{k+1})$$

- If $x^{k+1} = x^k$ stop

- [-] Condition (I) is easy to satisfy (at least if x^k is not a critical point)
- [-] Condition (II) is equivalent to solve exactly the DCA's subproblem

How to define trial points x^{k+1} - I?

$$(I) \quad f_1(x^{k+1}) - [f_2(x^k) + \langle g_2^k, x^{k+1} - x^k \rangle] \leq f_1(x^k) - f_2(x^k) \quad \text{and} \quad x^{k+1} \neq x^k$$

or

$$(II) \quad g_2^k \in \partial f_1(x^{k+1}) + N_{Y(x^k)}(x^{k+1})$$

In our application:

- c_1 is linear, implying that $Y(x^k) := \{x \in X : c_1(x) \leq c_2(x^k) + \langle s_2^k, x - x^k \rangle\}$ is a polyhedron
- f_1 is a nonsmooth function

How to define trial points x^{k+1} - II ?

An implementable manner to satisfy (I) or (II) consists in constructing a cutting-plane model $\check{f}_1^{\nu}(x)$ of $f_1(x)$:

Inner loop

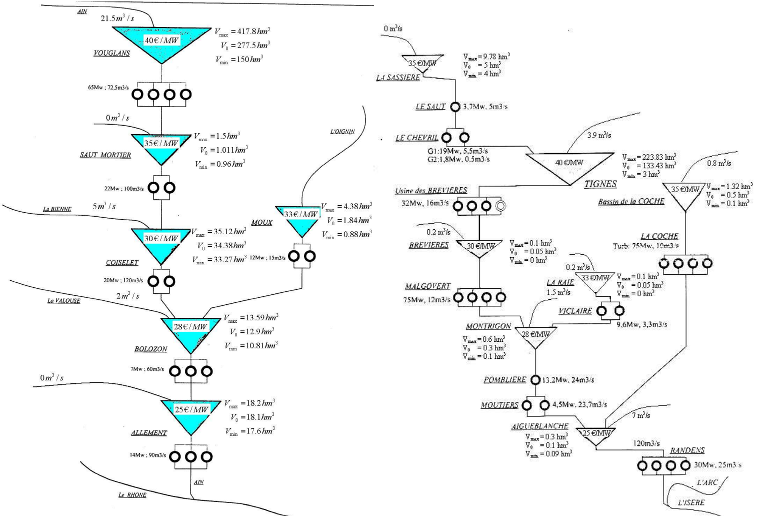
- Given $z^0 \in Y(x^k)$, choose $\check{f}_1^0 \leq f_1(x)$
- For $\nu = 0, 1, \ldots,$ compute

$$z^{\nu+1} \in \left\{ \begin{array}{lll} \arg\min_x & \check{f}_1^{\nu}(x) - [f_2(x^k) + \langle g_2^k, x - x^k \rangle] & \text{for an arbitrary } g_2^k \in \partial f_2(x^k) \\ \text{s.t.} & c_1(x) \leq c_2(x^k) + \langle s_2^k, x - x^k \rangle & \text{for an arbitrary } s_2^k \in \partial c_2(x^k) \\ & x \in X & \end{array} \right.$$

- Compute $(f_1(z^{\nu+1}), g_1^{\nu+1})$ and update the model $\check{f}_1^{\nu+1}$
- Test (I) with x^{k+1} replaced by $z^{\nu+1}$
  If it is satisfied, stop and define $x^{k+1} = z^{\nu+1}$

Preliminary numerical experiments

- We consider a set of hydro-thermal generation assets
- Two cascade reservoir systems: the Ain and Isère hydro valleys



Preliminary numerical experiments

- The power output of these assets is considered to be continuous between 0 and some maximum power level P_{mx}
- The time horizon is of 2 days with a $\Delta t = 2$ hour time step
- There are some ramping rates s (MW/h), such that any adjacent power levels may differ no more than $s\Delta t$
- Turbines and pumps connect a set of reservoirs
- Constraints involve preservation of mass (flow equations), physical bounds on contents in each reservoir and the amount of energy generated by turbining water
- There are 10 Thermal plants, being 1 of them on control of the local actor
- The demand side management tools are considered of the form of load displacement. This schematically models the option to recharge an electrical vehicle at a better moment in time rather than at peaking hours

Preliminary numerical experiments

$$\begin{cases} \min_x & f_1(x) - f_2(x) \\ \text{s.t.} & c_1(x) - c_2(x) \leq 0 \\ & x \in X \end{cases}$$

- f_1 is a polyhedral convex map known through a black box procedure requiring us to solve several *second-stage* optimization subproblems (N LPs)
- $f_2 = 0$
- c_1 is linear mapping known through a black-box requiring us to solve a single (linear) lower level optimization problem (1 LP)
- The dimension of x is 1322
- X is a polyhedron containing 1303 constraints

Computing f_1 is more expensive than computing c_2

Preliminary numerical experiments

- Our implementation, can due to simplicity of c_1, account explicitly for the set $Y(x^k)$, but needs to build a cutting plane model for f_1 and thus tries to trigger condition (I)

$$(I) \quad f_1(x^{k+1}) - [f_2(x^k) + \langle g_2^k, x^{k+1} - x^k \rangle] \leq f_1(x^k) - f_2(x^k) \quad \text{and} \quad x^{k+1} \neq x^k$$

We compared two solvers:

- DCA - An implementation of DC algorithm, solving subproblems up to optimality
- IDCA - Our proposal, solving subproblems inexactly

Both solvers are ensured to find critical points of general DC programming problems

	DCA	IDCA
# oracle calls for f_1	1382	989
# oracle calls for c_2	12	191
# iterations	12	191
CPU time	1710	679

- IDCA provided more than 60% of CPU time reduction
- Both solvers found the same critical point

Evaluation of the obtained decision

- In order to evaluate the quality of the obtained critical point, we have also solved a coordinated two-stage optimization problem wherein the global actor has full control over all assets
- In terms of optimal cost, the bilevel solution (critical point) is found to be only 0.4 % more costly than a coordinated two-stage solution
- As the two-stage solution yields a valid lower bound on the objective function value of the bilevel solution, we can conclude on the near “global" optimality of the found solution
- It has been observed in the literature that this class of algorithms does provide a “near" global solution quite often. We observe the same thing here

Future steps

We aim to

- compare numerically, in this application, DC algorithms against specialized bilevel algorithms
- add a probabilistic constraint to the DC problem
  - Since probabilistic functions can be approximated by Monte-Carlo simulation and DC decomposition, a DC programming formulation for the resulting Chance-Constrained DC problem is also possible
- include binary variables modeling "on/off " of power units
  - Binary constraints can be also modeled as a DC constraint:

$$z \in \{0, 1\} \quad \Leftrightarrow \quad z \in [0, 1] \quad z - z^2 \leq 0$$

Summary

In this talk we have discussed energy bilevel optimization problems and DC based methods for solving them. More:

- W. van Ackooij and W. de Oliveira. DC programming techniques with inexact subproblems' solution for general DC programs.
  Submitted manuscript: preprint available `http://www.oliveira.mat.br/publications`, pages 1–27, 2017

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