Modelling uncertainties in short-term operational planning optimization

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Key-points of the project

Evolution of[1]:

- electric power generation and transmission
  - use of renewable and distributed generators
  - end of a top-down electrical paradigm
- electricity usage
  - electric vehicles
  - consumption of self produced electricity
- social, political, technological evolution and requirements

Which imply:

- an increasing use of the distribution network
- an increase in the uncertainties on production and consumption[2]

[1] E-cube. "Étude Sur La Valeur Des Flexibilités Pour La Gestion Et Le Dimensionnement Des Réseaux De Distribution". In: (2016), pp. 1–102.

[2] Bhargav Prasanna Swaminathan. "Operational Planning of Active Distribution Networks - To cite this version : HAL Id : tel-01690509 Gestion prévisionnelle des réseaux actifs de distribution – relaxation convexe sous incertitude". In: (2018).

Current state of grid management

Operators face different challenges:

- little visibility on grid status
- no tools to calculate electric characteristics
- increasing variability of power transits $\Longrightarrow$ difficulty to anticipate network's behavior

Need of a tool to **support decision making**, in order to:

- validate (or not) a line disconnection
- have criteria when selecting levers to activate
- provide better insight in TSO/DSO communications
- always maintain our network within its limits

Grid-model and optimization problem

1. define a criteria to optimize
2. choose what to model
3. how to conciliate with a "solvable" maths model?
4. how to solve our model?

Traditional Optimal Power Flow:

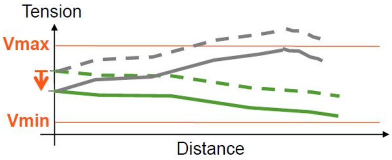
$$
\begin{aligned}
&\min f(x) \\
&\text{s.t. } P_i(V,\delta) = P_i^G - P_i^L && \forall i \in \mathbf{N} \\
&Q_i(V,\delta) = Q_i^G - Q_i^L && \forall i \in \mathbf{N} \\
&P_i^{G,min} \leq P_i^G \leq P_i^{G,max} && \forall i \in \mathbf{G} \\
&Q_i^{G,min} \leq Q_i^G \leq Q_i^{G,max} && \forall i \in \mathbf{G} \\
&V_i^{min} \leq V \leq V_i^{max} && \forall i \in \mathbf{N} \\
&\delta_i^{min} \leq \delta \leq \delta_i^{max} && \forall i \in \mathbf{N}
\end{aligned}
$$

Several key-words related to Operational Planning:

1. Unit-commitment / Economic dispatch
2. Security-constrained unit-comitment / Security-constrained economic dispatch
3. Add uncertainties

Levers to consider

1. Change value of imposed voltage at the "entry point"



2. Production modulation through contracts

   3 different types of contracts

3. Direct-load control through contracts

   batteries but **mainly** electric charging points

4. Access to an energy market
5. Topology changes
6. Use of capacitor banks

   impact on reactive power

7. Study impact of defaulting on supply

Current model - simplified version

$$\min_{x \in X} \quad c^T p^\delta$$

s.t.

$$I_{i,j} = Z_{i,j}(V_j - V_i) \qquad \forall (i,j) \in \mathbf{A}$$ **Ohm's Law**

$$S_{i,j} = \frac{1}{2}(V_i + I_{i,j})^2 - (V_i^2 + I_{i,j}^2) \qquad \forall (i,j) \in \mathbf{A}$$ **Defintion of Power**

$$S_j = \sum_{k:j \to k} S_{j,k} + Y_j V_j^2 + \sum_{k:k \to j} Z_{k,j} I_{k,j}^2 - \sum_{k:k \to j} S_{k,j} \qquad \forall j \in \mathbf{N}$$ **Power balance**

$$v_i \le \underline{v_i} \le v_i \qquad \forall i \in \mathbf{N}$$ **Voltage limits**

$$I_{i,j} \le \underline{I_{i,j}} \le I_{i,j} \qquad \forall (i,j) \in \mathbf{A}$$ **Transit limits**

$$p_i = p_i^g - p_i^l \qquad \forall i \in \mathbf{N}$$

$$p_i^g = \sum_{h \in \mathbf{B}_i^g} p_i^{\phi+,h} + \sum_{h \in \mathbf{B}_i^g} p_i^{\delta+,h} \qquad \forall i \in \mathbf{N}$$ **Power decomposition**

$$p_i^l = \sum_{h \in \mathbf{B}_i^l} p_i^{\phi-,h} + \sum_{h \in \mathbf{B}_i^g} p_i^{\delta-,h} + \sum_{h \in \mathbf{B}_i^l} p_i^{\nu,h} \qquad \forall i \in \mathbf{N}$$

$$0 \le p_i^{\delta-,h} \le p_i^{\phi+,h} - \underline{p_i^{g,h}} \qquad \forall h \in \mathbf{B}_i(NC1)$$

$$0 \le p_i^{\delta-,in,h} + p_i^{\delta-,out,h} = p_i^{\delta-,h} \qquad \forall h \in \mathbf{B}_i(NC2)$$ **Enforcing contracts' laws**

$$p_i^{\delta-,h} \le p_i^{\phi+,h} \qquad \forall h \in \mathbf{B}_i(NC2)$$

$$p_i^{\delta+,h} = p_i^{\delta-,h} = 0, \qquad \forall h \in \mathbf{B}_i(OC)$$

$$p^\nu \ge 0$$

Our problem' characteristics

In our particular case:

- non-linear (*e.g.* electric losses)
- non-convex (*e.g.* integer variables, possibly trigonometric functions)
- we want to include uncertainties

$\Longrightarrow$ a non trivial optimization problem

In case you would be interested in some relevant references:

- DTU Summer School lectures 2018
- Steven Low's work[3]

[3] Masoud Farivar and Steven H. Low. "Branch Flow Model: Relaxations and Convexification (Parts I, II)". In: (2012). ISSN: 0885-8950. DOI: 10.1109/TPWRS.2013.2255317. arXiv: 1204.4865.

Objectives and some statistics on our datasets

Some characteristics in our project:

- short-term optimization: D-4 to 30 minutes before
- time-step: 30 minutes
- HTA network: from the secondary of the transformer till HTA/BT transformer
- 3 types of contracts with producers
- direct-load control
- voltage control at the secondary of transformer
- electric vehicles - charging points

On a 32 bus network, in a Branch-Flow model, considering only producers as a lever:

- 24480 constraints
- 8304 variables

Introduction to *difference of convex* programming

Difference of convex structure: "the common underlying mathematical structure of virtually all nonconvex optimization problems"[4]

Definition

A function f is said to be *convex-concave* or a *difference of convex* on a convex set $\Omega \subset \mathbb{R}^n$ if exist two convex functions g and h such that:

$$f(x) = g(x) - h(x), \ \forall x \in \Omega$$

Properties

All functions of $\mathcal{C}^2(\mathbb{R}^n)$ *functions are convex-concave on convex compact sets of* $\mathbb{R}^n$
The set of convex-concave functions is dense in the set of continuous functions.

[4] Hoang Tuy. *Convex Analysis and Global Optimization*. Vol. 110. 2016. ISBN: 978-3-319-31482-2. DOI: 10.1007/978-3-319-31484-6. URL: http://link.springer.com/10.1007/978-3-319-31484-6.

Introduction to *difference of convex* algorithm

So we define a *Difference of Convex Program* as:

$$\boxed{\begin{array}{ll} \min\limits_{x \in X} & f_0(x) \\ s.t. & f_i(x) \leq 0, \;\; i \in [\![1, m]\!] \end{array}}$$

with $f_i = g_i - h_i, \;\; i \in [\![0, m]\!]$ *D.C.* functions and $X \subset \mathbb{R}^n$ a closed convex set

Algorithms to solve a DCP:

- *Difference of Convex Algorithm*: comes from Fenchel' Duality[5]
- *Convex-Concave Procedure*: a local heuristic that utilizes tools from convex optimization[6]

[5] Hoang Tuy. *Convex Analysis and Global Optimization*. Vol. 110. Springer Optimization and Its Applications. Cham: Springer International Publishing, 2016, pp. XII, 340. ISBN: 978-3-319-31482-2. DOI: 10.1007/978-3-319-31484-6. URL: http://link.springer.com/10.1007/978-3-319-31484-6.

[6] Thomas Lipp and Stephen Boyd. "Variations and extension of the convex–concave procedure". In: *Optimization and Engineering* 17.2 (2016), pp. 263–287. ISSN: 15732924. DOI: 10.1007/s11081-015-9294-x.

And what about uncertainties?

Continuous variables with sources of uncertainties:
Producers' production, loads' consumption

A generic way to model a constraint guaranteeing feasibility "as much as possible":[7]

$$\mathbb{P}[f(x,\xi) \leq 0] \geq p$$

with:

- x decision vector
- ξ random vector
- p probability level
- $f(x,\xi) \leq 0$ a finite system of inequalities

[7] Wim van Ackooij et al. "Chance Constrained Programming and Its Applications to Energy Management". In: *Stochastic Optimization - Seeing the Optimal for the Uncertain* June (2011). DOI: 10.5772/15438. URL: http://www.intechopen.com/books/stochastic-optimization-seeing-the-optimal-for-the-uncertain/chance-constrained-programming-and-its-applications-to-energy-management.

Stochastic extension

In our particular case, the optimization problem will be:

$$\begin{aligned} \min \quad & g_0(x) - h_0(x) & & & (1) \\ \text{s.t.} \quad & \mathbb{P}[g_i(x,\xi) - h_i(x,\xi) \leq 0] & \geq p, & \quad i \in \mathbf{I}_1 & (2) \\ & g_i(x,\xi) - h_i(x,\xi) & \leq 0, & \quad i \in \mathbf{I}_2 & (3) \\ & Ax & \leq 0, & & (4) \end{aligned}$$

Are there particular properties about constraints (2)?

Stochastic extension - DC approximation 1

Some interesting work to reformulate (2) in a *D.C.* way:

- **Approximation of the indicator function**

Recall we have:

$$\mathbb{P}[h(x,\xi) - g(x,\xi) \geq 0] = \mathbb{E}[\mathbf{1}_{[0,+\infty)}(g(x,\xi) - h(x,\xi))]$$

and we approximate $x \mapsto \mathbf{1}_{[0,+\infty)}(x)$ by ζ_t:

$$\zeta_t(z) = \frac{\max(z+t,0)}{t} - \frac{\max(z,0)}{t}$$

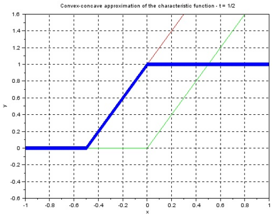


Figure: ζ_t function with $t = \frac{1}{2}$

Stochastic extension - DC Approximation 2

- **A mixed-binary relaxed-approach**

Step 1: Sampling and selection of scenarios

Let $(\xi^i)_{i\in\mathbb{I}}$ be a sample of scenarios with probabilities $(\pi^i)_{i\in\mathbf{I}_1}$

$$\mathbb{P}[g_i(x,\xi)-h_i(x,\xi)\leq 0]\geq p \approx \begin{cases} [g_i(x,\xi^j)-h_i(x,\xi^j)]z_j \leq 0, \forall j\in\mathbf{I} \\ \pi^T z \geq p \\ z_j\in\{0,1\}\ \forall j\in\mathbf{I} \end{cases}$$

Step 2: continuous relaxations ($t>0$)

$$\begin{aligned} z_j \in \{0;1\} \rightarrow & \qquad z_j\in[0;1] \\ [g_i(x,\xi^j)-h_i(x,\xi^j)]z_j \leq 0 \rightarrow & \qquad z_j \leq e^{-\frac{1}{t}[g_i(x,\xi^j)-h_i(x,\xi^j)]} \end{aligned} \tag{5}$$

Step 3: using the log and *max* function in (5)

$$\begin{aligned} z_j \leq e^{-\frac{1}{t}[g_i(x,\xi^j)-h_i(x,\xi^j)]} &\approx \qquad g_i(x,\xi^j)-h_i(x,\xi^j)+t\ \log(z_j)\leq 0, \forall j \\ &\approx \qquad \max_{j\in\mathbf{I}}\Big(g_i(x,\xi^j)-h_i(x,\xi^j)+t\log(z_j)\Big)\leq 0 \end{aligned}$$

Are probabilistic constraints convex-concave themselves?

Yes under some assumptions...[8]

Let's consider the following problem:

$$\begin{aligned} \min_{x \in X} \quad & f(x) \\ \text{s.t.} \quad & \mathbb{P}[g_1(x,\xi) - h_1(x,\xi) \leq 0] \geq p \\ & g_2(x) - h_2(x) \leq 0. \end{aligned}$$

Proposition

Assuming the following:

- $M(x) = \{z \in \mathbb{R}^m \colon g_1(x,z) - h_1(x,z) \leq 0\}$ *is convex*
- $\xi \in \mathbb{R}^m$ *is an elliptically symmetric random vector with "nice" properties*
- g_1 *and* h_1 *are convex in* x*;*
- g_1 *is convex in* ξ, h_1 *is concave in* ξ*;*

Then $\mathbb{P}[g_1(x,\xi) - h_1(x,\xi) \leq 0]$ *also is convex-concave*

[8] Wim van Ackooij and Jérôme Malick. *Eventual convexity of probability constraints with elliptical distributions*. 2018. DOI: 10.1007/s10107-018-1230-3.

Conclusion

Our objectives:

- provide a framework to fully study the impact of different grid models
- and of different solving methods (*e.g.* MINLP vs *D.C.* programming)
- have the ability to select different levers
- test several *D.C.* formulations
- provide sensitivity analysis
- investigate the stochastic side and its *D.C.* formulations
- in particular:
  what differences are there between uncertainties D-4 days *vs* h-30 minutes?

Conclusion

Thank you for your attention!

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